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B.M.S COLLEGE FOR WOMEN AUTONOMOUS

BENGALURU -560004

END SEMESTER EXAMINATION – APRIL/ MAY 2023

M.Sc. Mathematics – III Semester

FLUID MECHANICS

Course Code: MM304T **QP Code: 13004**
Duration: 3 Hours **Max. Marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. a) Define a Cartesian tensor of order N and explain any three properties of cartesian tensor.
   b) State and prove divergence theorem for a tensor field. **(8+6)**

2. a) For a material body in motion the displacement field is given by $u_1 = 0, u_2 = x_2 - {}^1/_2\,(x_2 + x_3)e^{-t} - {}^1/_2\,(x_2 - x_3)e^{t}, u_3 = x_3 - {}^1/_2\,(x_2 + x_3)e^{-t} + {}^1/_2\,(x_2 - x_3)e^{t}$ then find the velocity and acceleration fields in the material and spatial form.
   b) Establish the Reynolds transport formula and hence deduce the expression for the rate of change of a material volume. **(8+6)**

3. a) Explain the significance of equation of continuity and hence derive the field equation for conservation of mass.
   b) State and prove Kelvin's circulation theorem and hence establish the permanence of irrotational motion. **(7+7)**

4. a) Distinguish between non-viscous and viscous fluids. Also, find the pressure distribution in an incompressible non-viscous fluid moving under the earth's gravitational field with the velocity $\vec{q} = \nabla\phi$ where $\phi = x^3 - 3xy^2$.
   b) If the external forces are conservative and density is a function of pressure p only, then Prove that $\frac{d}{dt}\left(\frac{W}{\rho}\right) = \left(\frac{W}{\rho}.\nabla\right) q$. Hence deduce that $\vec{w}/_\rho$ = constant for a travelling fluid element. **(7+7)**

5. a) Derive the Navier – Stokes equation for a viscous fluids.
   b) Obtain the velocity distribution for the plane Poiseuille flow and show that the maximum velocity occurs in the middle of the channel. **(6+8)**

6. a) Show that the velocity distribution for Stokes first problem is $u(z,t) = U[1 - \mathrm{erf}(\eta)]$ where quantities have their usual meaning.
   b) Explain energy dissipation due to viscosity and establish the necessary expression for the same. **(8+6)**

7. a) Show that $u = 2Axy, v = A(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Dtermine the stream function.

   b) Define the source, sink and doublet. Obtain the complex potential for the doublet. **(7+7)**

8. a) For a two-dimensional flow field given by $\psi = xy,$ show that the flow is irrotational. Also, verify that ψ and ϕ satisfy the Laplace equation and find the velocity potential, streamlines and potential lines.

   b) State and prove Milne-Thomson circle theorem. **(7+7)**

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